AQA

A-level
MATHEMATICS
7357/3

Paper 3

Mark scheme

June 2021

Version: 1.0 Final Mark Scheme



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.2	B1	$(-1, \pi)$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	–2
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	1.1b	B1	$6x$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Express at least one term correctly using binomial expansion **PI** by correct value of p or q or expression for x^8 and x^9 Condone sign error for –3	1.1a	M1	$(2x)^{10} + {}^{10}C_1(2x)^9(-3) + {}^{10}C_2(2x)^8(-3)^2$ $1024x^{10} - 15360x^9 + 103680x^8$
	Obtains correct value of p or q **PI** in the expression may be unsimplified	1.1a	M1	
	Completes argument with correct values of p and q to show the required result	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Deduces the constant term comes from $(2x)^5\left(\pm\frac{3}{x}\right)^5$ **PI** by $(\ldots x)^5$ $(\ldots x)^{-5}$	2.2a	M1	${}^{10}C_5 \times 2^5 \times (-3)^5 = -1959552$
	Obtains –1959552	1.1b	A1	
	Subtotal		**2**	

	Question Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(i)	Uses formula correctly for area of sector	1.1a	M1	$A=\frac{1}{2}\times 5^2\times 0.7$ $=8.75\,\text{m}^2$
	Obtains 8.75 Condone incorrect or missing units	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(ii)	Uses formula for arc length	1.1b	B1	$P=5\times 0.7+2\times 5$ $=13.5$ $Cost=13.5\times 1.80$ $=£24.30$
	Obtains the perimeter by adding twice the radius to their arc length and multiplies their perimeter by 1.80	3.1b	M1	
	Obtains correct cost £24.30 **CAO**	3.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(i)	Forms at least one correct equation for area or perimeter May be embedded in the formulae for C	3.3	M1	$P=r\theta+2r$ $\frac{1}{2}r^2\theta=20$ $\Rightarrow\theta=\frac{40}{r^2}$ $P=\frac{40}{r}+2r$ $C=\frac{40\times 1.8}{r}+2\times 1.8r$ $=\frac{72}{r}+\frac{18}{5}r$ $=\frac{18}{5}\left(\frac{20}{r}+r\right)$
	Eliminates θ from two fully correct equations for area and perimeter to obtain an expression for P in terms of r	3.1b	A1	
	Completes argument to show the required result Accept 3.6 for $\frac{18}{5}$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(ii)	Recognises the use differentiation in the model **PI** if $\frac{dC}{dr}$ seen	3.4	B1	$C = \frac{72}{r} + \frac{18}{5}r$ $\frac{dC}{dr} = -\frac{72}{r^2} + \frac{18}{5}$ Minimum occurs when $\frac{dC}{dr}$ =0 $-\frac{72}{r^2} + \frac{18}{5} = 0$ r^2 = 20 $r = \sqrt{20} \approx 4.472..$ Hence $r \approx 4.5$ $\frac{d^2C}{dr^2} = \frac{144}{r^3}$ When $r = \sqrt{20}$, $\frac{d^2C}{dr^2} > 0$ Therefore minimum at $r \approx 4.5$
	Differentiates given model with at least one term correct Condone sign error **OE**	1.1b	M1	
	Explains that a minimum/stationary/turning point occurs when $\frac{dC}{dr} = 0$	2.4	E1	
	Solves $\frac{dC}{dr} = 0$ to find correct exact value or decimal value for r to at least two decimal places	1.1b	A1	
	Uses a gradient test or second derivative or sketches graph to determine nature of stationary point Completes argument to show minimum occurs when $r \approx 4.5$ Must have shown $r \approx 4.5$ in previous step	2.1	R1	
	Subtotal		**5**	

	Question Total		**13**	

Q	Marking instructions	AO	Marks	Typical solution
6	Begins to solve the problem using an appropriate technique eg factorising or grouping terms in numerator or writing $y = \sqrt{x}$ **PI** if $2 + x$ or $25 - x$ or $5 - x^{1/2}$ seen or multiplies by $\frac{5+\sqrt{x}}{5+\sqrt{x}}$	3.1a	M1	$\frac{10+5x-2x^{\frac{1}{2}}-x^{\frac{3}{2}}}{5-\sqrt{x}} \times \frac{5+\sqrt{x}}{5+\sqrt{x}}$ $= \frac{50+25x-10x^{\frac{1}{2}}-5x^{\frac{3}{2}}+10\sqrt{x}+5x\sqrt{x}-2x-x^{\frac{3}{2}}\sqrt{x}}{25-x}$ $= \frac{50+23x-x^2}{25-x}$ $= \frac{(25-x)(2+x)}{25-x}$ $= 2+x$
	Obtains one correct common factor in numerator eg $2 + x$ or $25 - x$ or $5 - x^{1/2}$ or expands numerator condone one error may be unsimplified	1.1a	M1	
	Obtains second correct common factor in numerator or obtains correct simplified numerator and denominator **PI** in long division	1.1a	M1	
	Completes manipulation by cancelling common factor to obtain $2 + x$	1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Obtains the correct volume **AWRT** 29 Condone incorrect or missing units	1.1b	B1	$W_2 = 29.4$
	Subtotal		**1**	

Q	Marking instructions	AO	Mark	Typical solution
7(b)	States $A = 30$ **PI** by building up of a sequence to three terms or $W_n = 30 \times 0.98^{n-1}$ seen	1.1b	B1	W_n is the nth term of a geometric sequence, a 2% reduction gives a common ratio of 0.98 A = 30
	Explains W_n is (the nth term of) a geometric sequence explaining that a 2% reduction gives a common ratio of 0.98 **PI** by building up of a sequence to three terms	3.3	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Mark	Typical solution
7(c)	Uses geometric model with their value of A substituted to find S_{15}	3.4	M1	$S_{15} = \dfrac{30\left(1-0.98^{15}\right)}{1-0.98}$ $= 392$
	Obtains their correct value of S_{15} **FT** their value of A Condone unrounded answers	1.1b	A1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Mark	Typical solution
7(d)	Uses sum to infinity formula with their value of A substituted	3.4	M1	$S_\infty = \frac{30}{1-0.98}$ $= 1500$
	Obtains their correct value of sum to infinity	1.1b	A1F	1.5 + 4 = 5.5 litres
	Obtains 5.5 litres **CAO** Accept answer in litres or millilitres	3.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Mark	Typical solution
7(e)	Explains that the model used assumes the drips continue indefinitely which is unrealistic	3.5b	E1	The sum to infinity was used but this assumes there are infinite drips, but they have stopped
	States a relevant environmental factor eg water has evaporated or wind affected water level or water consumed by animals	3.5a	E1	Water will evaporate over several hours
	Subtotal		**2**	

	Question Total		**10**	

Q	Marking instructions	AO	Mark	Typical solution
8	Uses integration by parts with $u = x$ and $v' = \cos x$ **PI** by $x\sin x + \cos x$	3.1a	B1	$u = x \qquad u' = 1$ $v' = \cos x \qquad v = \sin x$ $\int x\cos x\,\mathrm{d}x = x\sin x - \int \sin x\,\mathrm{d}x$ $= x\sin x + \cos x$ $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} x\cos x\,\mathrm{d}x = \left[x\sin x + \cos x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \frac{\pi}{3}\sin\frac{\pi}{3} + \cos\frac{\pi}{3} - \left(\frac{\pi}{4}\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)$ $= \pi\frac{\sqrt{3}}{6} + \frac{1}{2} - \left(\pi\frac{\sqrt{2}}{8} + \frac{\sqrt{2}}{2}\right)$ $= \left(\frac{4\sqrt{3} - 3\sqrt{2}}{24}\right)\pi + \left(\frac{1-\sqrt{2}}{2}\right)$
	Applies integration by parts formula correctly by substituting their u, u', v and v' **PI** by $x\sin x + \cos x$	1.1a	M1	
	Obtains $x\sin x + \cos x$ **CAO**	1.1b	A1	
	Substitutes limits correctly into their integrated expression **PI** by correct a and b	1.1a	M1	
	Uses correct exact value for any one of $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ or $\cos\frac{\pi}{3} = \frac{1}{2}$ or $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ or $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ **PI** by correct a or b	1.2	B1	
	Obtains correct exact values of a and b **ACF** Ignore if 0.14(…) seen subsequently	1.1b	A1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Differentiates $f(x)$ at least one correct term May be unsimplified	1.1a	M1	$f'(x) = 4x^3 + 15x^2$ $f''(x) = 12x^2 + 30x$
	Obtains $f''(x) = 12x^2 + 30x$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(ii)	Substitutes $x = -\frac{15}{4}$ into their $f''(x)$ or uses gradient test both sides of $x = -\frac{15}{4}$	1.1a	M1	$f''\left(-\frac{15}{4}\right) = 12\left(-\frac{15}{4}\right)^2 + 30\left(-\frac{15}{4}\right)$ $= \frac{225}{4} > 0$ Hence there is a minimum at $x = -\frac{15}{4}$ $f''(0) = 0$ $f''(1) = 12 + 30 > 0$ and $f''(-1) = 12 - 30 < 0$ hence point of inflection at $x = 0$
	Completes rigorous justification for minimum at $x = -\frac{15}{4}$ This must be correctly deduced using shape of graph or $f''\left(-\frac{15}{4}\right) = \frac{225}{4} > 0$	2.1	R1	
	Substitutes two values either side of $x = 0$ into their $f''(x)$ or uses gradient test both sides of $x = 0$ or argues using the shape of a quartic curve with two stationary points	1.1a	M1	
	Completes rigorous justification for point of inflection at $x = 0$ This must be correctly deduced using the shape of the graph or a completely correct test both sides of the point Other explanation eg quartic with two stationary points, one of the points must be a point of inflection	2.2a	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Deduces $x > -\frac{15}{4}$ **OE** Condone use of ′ ≥’	2.2a	B1	$x > -\frac{15}{4}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(i)	Deduces the transformation is a reflection in the y-axis **OE**	2.2a	B1	Reflection in the y-axis
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(ii)	Deduces $x > \frac{15}{4}$ Condone use of ′ ≥’ **FT** their answer in part (b) only if their value in (b) is negative	2.2a	B1F	$x > \frac{15}{4}$
	Subtotal		**1**	

	Question Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
10	Ticks correct box	2.2b	R1	Definitely incorrect
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	1.1b	B1	0.36
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Explains how to enumerate population using valid numbering stating range used	2.4	E1	Give each name a number from 1 to 8000 $\frac{8000}{100} = 80$ Randomly select a number between 1 and 80 and select every 80th person on the register from the first person selected
	Calculates 8000÷100 or 80 seen	1.1b	B1	
	Explains that they randomly select the first person from the first 80 people and thereafter every subsequent 80th person should be selected	2.4	E1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)(i)	Calculates correct value of mean **AWRT** 149	1.1b	B1	Mean = 148.6 Standard deviation = 17.8
	Calculates correct value of standard deviation **AWRT** 17.8 Accept **AWRT** 18.5	1.1b	B1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)(ii)	Calculates either their mean + 2×standard deviation or their mean – 2×standard deviation	1.1b	M1	$148.6 + 2 \times 17.8 = 184.2$ $148.6 - 2 \times 17.8 = 113$ $192 > 184.2$ 192 is the only outlier
	Deduces that the CO2 value of 192 is the only outlier – must make a clear comparison and have both lower and upper outlier boundaries **FT** their mean and standard deviation	2.2a	R1F	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(b)	Explains that the 0 value is an error because every car has a mass or there is a driver mass	2.2b	E1	The 0 value is an error because every car has a mass The blank cell may not be an error as not all particulate emissions are recorded in the LDS
	Explains that the blank cell may not be an error as the LDS only has particulate emissions recorded for some cars	2.2b	E1	
	Subtotal		**2**	

	Question Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Uses $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ with 0.1 substituted correctly or draws a Venn diagram with 0.1 in the correct region	3.1a	B1	$P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $P(A\cup B)=0.8$ $0.8 = P(A) + 2P(A) - 0.1$ $3P(A) = 0.9$ $P(A) = 0.3$
	Uses $P(A\cup B)=0.8$ in the equation **PI** by showing at least three of 0.2, 0.1, x – 0.1 or $2x$ – 0.1 in the correct regions on the Venn diagram	1.1b	B1	
	Substitutes for $P(B)$ to form an equation to find $P(A)$ **PI** by correct answer for $P(A)$ or shows all of 0.2, 0.1, x – 0.1 and $2x$ – 0.1 in the correct regions on the Venn diagram	1.1a	M1	
	Obtains $P(A) = 0.3$	1.1b	A1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(b)	Uses conditional probability formula with 0.1 and their $P(A)$ substituted correctly	3.1a	M1	$P(A\cap B)=P(A)\times P(B\mid A)$ $P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}$ $=\dfrac{0.1}{0.3}$
	Obtains correct answer **FT** their $P(A)$ if 0.1 < $P(A)$ < 1 Allow $0.\dot{3}$ but not 0.33(…) for $\frac{1}{3}$	1.1b	A1F	$=\dfrac{1}{3}$
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(c)	Deduces that A and B are **not independent** by comparing with 0.1 or shows $P(B)=0.6\neq P(B\mid A)$	2.2a	R1	Not independent as $P(A)\times P(B)=0.3\times 0.6=0.18$ $\neq P(A\cap B)$ because P(A∩B) = 0.1
	Subtotal		**1**	

	Question Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
15	States both hypotheses correctly for two-tailed test Accept **population mean** for μ	2.5	B1	X = times to solve in minutes $H_0: \mu = 65$ $H_1: \mu \neq 65$
	Calculates mean of the sample **PI** in equation	1.1b	B1	$\overline{x} = \frac{6780}{100} = 67.8$
	Formulates the test statistic or uses the correct distribution of their sample mean **PI** by correct test statistic value or calculates probability or identifies acceptance region Condone $65 - 67.8$	3.3	M1	$\text{Test statistic} = \frac{67.8 - 65}{11.3/\sqrt{100}}$ $= 2.48$
	Obtains the correct value of the test statistic [2.47, 2.5] or obtains the correct probability [0.0066, 0.007] or [0.0132, 0.014] or obtains the correct acceptance region of [62.3, 67.7]	1.1b	A1	Critical value = 2.33 2.48 > 2.33 Reject H_0 There is sufficient evidence at the 2% level to suggest that mean escape time has changed
	Compares their value of test statistic [2.47, 2.5] with their critical value 2.33 Allow critical value [-4, 4] except ±0.02 or ±0.01 or compares their probability [0.0066, 0.007] with 0.01 or compares their probability [0.0132, 0.014] with 0.02 or compares their sample mean 67.8 with their acceptance region [62.3, 67.7]	1.1b	M1	
	Compares correct values and correctly infers H_0 is rejected **CSO** Allow reference to H_1	2.2b	A1	
	Concludes correctly in context that there is **sufficient evidence** to suggest that the **mean** escape **time** has **changed** **CSO**	3.2a	R1	
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	Substitutes x values into probability function to obtain at least three correct expressions in terms of c	1.1a	M1	$\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline x & 0 & 1 & 2 & 3 & 4 \\ \hline \mathrm{P}(X=x) & 7c & 5c & 3c & c & k \\ \hline \end{array}$ $7c + 5c + 3c + c + k = 1$ $16c + k = 1$
	Completes rigorous argument by obtaining five correct expressions and summing them to 1 to obtain required result **OE**	2.1	R1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
16(b)	Forms a second equation using their expressions for $P(X=3)$ and $P(X=4)$	1.1a	M1	$c+k=\frac{5}{8}$ $c=\frac{1}{40} \quad k=\frac{3}{5}$
	Obtains $c=\frac{1}{40}$ and $k=\frac{3}{5}$ **OE**	1.1b	A1	
	Subtotal		**2**	

	Question Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)	States one correct binomial assumption in context	3.5b	E1	The event of James winning one game is independent of him winning another game The probability of James winning remains constant at 0.6 from game to game
	States a second correct binomial assumption in context eg each time he plays he can only win or not win or the number games is fixed or winning one game is independent of him winning another game Condone omission of 0.6 from the statement	3.5b	E1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(b)	Obtains correct probability **AWRT** 0.11	3.1b	B1	0.111
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(c)	Calculates either $P(Y \leq 3) = 0.05476$ or $P(Y \leq 4) = 0.16623$ using the Binomial distribution or states $P(Y \geq 4) = 1 - P(Y \leq 3)$ or subtracts their stated value of $P(Y \leq 3)$ from 1	3.1b	M1	$P(Y \leq 3) = 0.05476$ $P(Y \geq 4) = 1 - P(Y \leq 3)$ $= 1 - 0.05476$ $= 0.94524$
	Obtains correct probability **AWFW** [0.94, 0.95]	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(d)	States both hypotheses correctly for a one-tailed test	2.5	B1	X = number of games won H_0: p = 0.6 H_1: p > 0.6 X ~B(15,0.6) P(X ≥ 12) = 1 – P($X \leq 11$) = 1 – 0.9094 = 0.0905 0.0905 > 0.05 so accept H_0 There is insufficient evidence to suggest that the probability of James winning the game has increased.
	Uses correct binomial model to obtain either P($X \leq 11$) or $P(X \leq 12)$ or $P(X \geq 13)$ or $P(X \geq 14)$ **PI** by critical region $X \geq 13$ *or* $X \geq 14$	3.3	M1	
	Obtains the correct probability for $P(X \geq 12)$ or obtains correct critical region $X \geq 13$	1.1b	A1	
	Evaluates binomial model by comparing their $P(X \geq 12)$ with 0.05 or Compares 12 with their critical region and makes their inference	3.5a	M1	
	Infer H_0 is not rejected **CSO** Allow reference to H_1	2.2b	A1	
	Concludes correctly in context that there is **insufficient evidence** to suggest that the **probability** of winning the **game** has **increased.**	3.2a	R1	
	Subtotal		**6**	

	Question Total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
18(a)(i)	States 0	1.2	B1	0
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
18(a)(ii)	Uses the normal distribution model to calculate P(X < 368) or shows $P(X > 368) = 1 - P(X < 368)$ **PI** by correct answer	3.1b	M1	$P(X > 368) = 0.87345$
	Obtains correct probability **AWRT** 0.87	1.1b	A1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
18(b)(i)	Explains that the 1.96 is obtained through inverse normal distribution function or shows 1.96 on a diagram **PI** if 1.959(…) seen	2.4	E1	Using inverse normal, the z- value is 1.95996398 for the area of 0.975 $P\left(Z < \frac{346-\mu}{\sigma}\right) = 0.975$
	Forms an equation with unknown μ and σ using standardised result and their z-value Accept z = (−4, 4) except ±0.975 Condone $\mu - 346$	3.1b	M1	$\frac{346-\mu}{\sigma} = 1.96$ Hence $346 - \mu = 1.96\,\sigma$
	Completes rigorous argument by forming a correct equation using 1.96 and rearranging the equation	2.1	R1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
18(b)(ii)	Obtains either z-value from inverse normal distribution Condone sign error **AWFW** [−1.1, −1.08]	1.1b	B1	$z = -1.08$ $\frac{336-\mu}{\sigma} = -1.08$ $336-\mu = -1.08\sigma$ $\sigma = 3.29$ $\mu = 340$
	Forms second equation with unknown μ and σ using standardised result and their z-value Accept z = (−4, 4) except ±0.14 Condone $\mu - 336$	1.1a	M1	
	Obtains correct value of σ **AWRT** 3.3 **ISW**	1.1b	A1	
	Obtains correct value of μ **AWRT** 340 **ISW**	1.1b	A1	
	Subtotal		**4**	

	Question Total		**10**	